# 4.9 Normal distribution and calculations\_P\_2

**1a.** *[2 marks]*

Timmy owns a shop. His daily income from selling his goods can be modelled as a normal distribution, with a mean daily income of 230. To make a profit, Timmy’s daily income needs to be greater than $1000.

Calculate the probability that, on a randomly selected day, Timmy makes a profit.

**1b.** *[3 marks]*

The shop is open for 24 days every month.

Calculate the probability that, in a randomly selected month, Timmy makes a profit on between 5 and 10 days (inclusive).

**2a.** *[6 marks]*

It is known that 56 % of Infiglow batteries have a life of less than 16 hours, and 94 % have a life less than 17 hours. It can be assumed that battery life is modelled by the normal distribution .

Find the value of  and the value of .

**2b.** *[2 marks]*

Find the probability that a randomly selected Infiglow battery will have a life of at least 15 hours.

**3a.** *[2 marks]*

The age, *L*, in years, of a wolf can be modelled by the normal distribution *L* ~ N(8, 5).

Find the probability that a wolf selected at random is at least 5 years old.

**3b.** *[3 marks]*

Eight wolves are independently selected at random and their ages recorded.

Find the probability that more than six of these wolves are at least 5 years old.

**4a.** *[2 marks]*

The random variable *X* has a normal distribution with mean *μ* = 50 and variance *σ*2 = 16 .

Sketch the probability density function for *X*, and shade the region representing P(*μ* − 2σ < *X* < *μ* + σ).

**4b.** *[2 marks]*

Find the value of P(*μ* − 2σ < *X* < *μ* + σ).

**4c.** *[2 marks]*

Find the value of *k* for which P(*μ* − *k*σ < *X* < *μ* + *k*σ) = 0.5.

**5.** *[6 marks]*

It is given that one in five cups of coffee contain more than 120 mg of caffeine.  
It is also known that three in five cups contain more than 110 mg of caffeine.

Assume that the caffeine content of coffee is modelled by a normal distribution.  
Find the mean and standard deviation of the caffeine content of coffee.

**6a.** *[2 marks]*

The times taken for male runners to complete a marathon can be modelled by a normal distribution with a mean 196 minutes and a standard deviation 24 minutes.

Find the probability that a runner selected at random will complete the marathon in less than 3 hours.

**6b.** *[2 marks]*

It is found that 5% of the male runners complete the marathon in less than  minutes.

Calculate .

**6c.** *[4 marks]*

The times taken for female runners to complete the marathon can be modelled by a normal distribution with a mean 210 minutes. It is found that 58% of female runners complete the marathon between 185 and 235 minutes.

Find the standard deviation of the times taken by female runners.

**7a.** *[2 marks]*

Packets of biscuits are produced by a machine. The weights , in grams, of packets of biscuits can be modelled by a normal distribution where . A packet of biscuits is considered to be underweight if it weighs less than 250 grams.

Given that  and  find the probability that a randomly chosen packet of biscuits is underweight.

**7b.** *[3 marks]*

The manufacturer makes the decision that the probability that a packet is underweight should be 0.002. To do this  is increased and  remains unchanged.

Calculate the new value of  giving your answer correct to two decimal places.

**7c.** *[2 marks]*

The manufacturer is happy with the decision that the probability that a packet is underweight should be 0.002, but is unhappy with the way in which this was achieved. The machine is now adjusted to reduce  and return  to 253.

Calculate the new value of .

**8a.** *[6 marks]*

A random variable  is normally distributed with mean  and standard deviation , such that  and .

Find  and .

**8b.** *[2 marks]*

Find .

**9.** *[6 marks]*

The heights of students in a single year group in a large school can be modelled by a normal distribution.

It is given that 40% of the students are shorter than 1.62 m and 25% are taller than 1.79 m.

Find the mean and standard deviation of the heights of the students.

**10a.** *[2 marks]*

A random variable  is normally distributed with mean 3 and variance .

Find .

**10b.** *[3 marks]*

Find .

**10c.** *[2 marks]*

If , find the value of .

**11a.** *[6 marks]*

A survey is conducted in a large office building. It is found that  of the office workers weigh less than  kg and that  of the office workers weigh more than  kg.

The weights of the office workers may be modelled by a normal distribution with mean  and standard deviation .

(i)     Determine two simultaneous linear equations satisfied by  and .

(ii)     Find the values of  and .

**11b.** *[1 mark]*

Find the probability that an office worker weighs more than  kg.

**11c.** *[2 marks]*

There are elevators in the office building that take the office workers to their offices.

Given that there are  workers in a particular elevator,

find the probability that at least four of the workers weigh more than  kg.

**11d.** *[3 marks]*

Given that there are  workers in an elevator and at least one weighs more than  kg,

find the probability that there are fewer than four workers exceeding  kg.

**11e.** *[3 marks]*

The arrival of the elevators at the ground floor between  and  can be modelled by a Poisson distribution. Elevators arrive on average every  seconds.

Find the probability that in any half hour period between  and  more than  elevators arrive at the ground floor.

**11f.** *[3 marks]*

An elevator can take a maximum of  workers. Given that  workers arrive in a half hour period independently of each other,

find the probability that there are sufficient elevators to take them to their offices.

**12a.** *[2 marks]*

The finishing times in a marathon race follow a normal distribution with mean 210 minutes and standard deviation 22 minutes.

Find the probability that a runner finishes the race in under three hours.

**12b.** *[2 marks]*

The fastest  of the finishers receive a certificate.

Find the time, below which a competitor has to complete the race, in order to gain a certificate.

**13a.** *[6 marks]*

Farmer Suzie grows turnips and the weights of her turnips are normally distributed with a mean of  and standard deviation of .

(i)     Calculate the percentage of Suzie’s turnips that weigh between  and .

(ii)     Suzie has  turnips to take to market. Find the expected number weighing more than .

(iii)     Find the probability that at least  of the  turnips weigh more than .

**13b.** *[6 marks]*

Farmer Ray also grows turnips and the weights of his turnips are normally distributed with a mean of . Ray only takes to market turnips that weigh more than . Over a period of time, Ray finds he has to reject  in  turnips due to their being underweight.

(i)     Find the standard deviation of the weights of Ray’s turnips.

(ii)     Ray has  turnips to take to market. Find the expected number weighing more than .

**14a.** *[2 marks]*

The wingspans of a certain species of bird can be modelled by a normal distribution with mean  cm and standard deviation  cm.

According to this model,  of wingspans are greater than cm.

Find the value of .

**14b.** *[3 marks]*

In a field experiment, a research team studies a large sample of these birds. The wingspans of each bird are measured correct to the nearest  cm.

Find the probability that a randomly selected bird has a wingspan measured as  cm.

**15.** *[5 marks]*

A student sits a national test and is told that the marks follow a normal distribution with mean 100. The student receives a mark of 124 and is told that he is at the  percentile.

Calculate the variance of the distribution.

**16.** *[5 marks]*

The weights, in kg, of one-year-old bear cubs are modelled by a normal distribution with mean  and standard deviation .

(a)     Given that the upper quartile weight is 21.3 kg and the lower quartile weight is 17.1 kg, calculate the value of  and the value of .

A random sample of 100 of these bear cubs is selected.

(b)     Find the expected number of bear cubs weighing more than 22 kg.

Printed for SANSKAR SCHOOL

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